

THE TRANSMISSION OF SUPPORT: A BAYESIAN RE-ANALYSIS

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Abstract

Crispin Wright's discussion of the notion of 'transmission-failure' promises to have important philosophical ramifications, both in epistemology and beyond. This paper offers a precise, formal characterisation of the concept within a Bayesian framework. The interpretation given avoids the serious shortcomings of a recent alternative proposal due to Samir Okasha.

1. Introduction As part of a wider agenda in both epistemology and philosophy of mind, Crispin Wright has now written at considerable length about what he calls the principle of 'transmission' of warrant over entailment (see for instance [Wright, 1985], [Wright, 2000], [Wright, 2002] and [Wright, 2003]). His characterisation of the principle is somewhat difficult to pin down in precise terms, but it would appear to amount to something like the following:

Transmission: For all propositions A , B and C , necessarily, if A is evidence for B and B entails C , then A is thereby evidence for C .

Triples of propositions $\langle A, B, C \rangle$ that satisfy both the antecedent and the consequent of the conditional are said to exhibit 'transmission-success'. Conversely, satisfaction of the antecedent but not the consequent results in 'transmission-failure'.¹

A number of brief clarifications are in order with respect to the above principle. First of all, Wright appears to use the term 'entailment' in the fairly liberal sense that A entails B iff the set of metaphysically possible worlds in which A is true is a subset of the set of metaphysically possible worlds in which B is true. I shall follow suit. Secondly, as Wright makes clear [Wright, 2000, p. 140], Transmission is distinct from the weaker

Closure: For all propositions B and C , necessarily, if one has evidence for B and B entails C , then one has evidence for C .

Specifically, Closure leaves open whether the evidence that supports the conclusion C does include the evidence A that supports the premise B (as required for transmission-success) or does not (as entailed by transmission-failure). Finally, Wright formulates

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both principles in terms of *known* entailment. I will ignore this qualification in what follows as, in my view, (i) it would needlessly complicate matters and (ii) the resulting principles do not thereby gain in plausibility.

Now according to Wright [Wright, 2000, p. 141], whilst Closure may seem intuitively true, it would seem, upon reflection at least, that Transmission is false. In [Wright, 1985, p. 433], he appears to tell us that the latter fails notably when the following is true:

Exception: The support that *A* gives to *B* is conditional on having independent reason to believe *C*.

The following set of propositions is plausibly offered as an example of transmission-failure in which Exception is satisfied [Wright, 2003, p. 342]:

Zebra: *A_Z*: The animal in the enclosure is a black & white–striped four-legged equine creature. *B_Z*: The animal in the enclosure is a zebra. *C_Z*: The animal in the enclosure is not a cleverly painted mule.

Intuitively, (i) *A_Z* is not evidence for *C_Z*, (ii) *A_Z* is evidence for *B_Z* only given independent evidence for *C_Z* and finally (iii) Closure holds—if one has evidence for *B_Z*, one has evidence for *C_Z*.

According to a number of authors, failures of transmission also feature in such philosophically important cases as (a) the arguments to external-world contingencies that figure in McKinsey-style reductios of semantic externalism [Wright, 2000], [Davies, 1998], [Davies, 2000], [Davies, 2003], (b) Moore’s argument against external-world skepticism [Wright, 2002], or again (c) Boghossian’s argument to a priori knowledge of logical principles [Ebert, 2005]. In contrast, instances in which Transmission is satisfied would include, for instance [Wright, 2002, p. 332]

Death Cap: *A_D*: 3 hours ago, Jones consumed a large risotto of death caps. *B_D*: Jones has absorbed a large quantity of amatotoxins. *C_D*: Jones will shortly die.

Again, intuitively, (i) *A_D* is evidence for *C_D*, (ii) *A_D* is evidence for *B_D* whether or not one has independent evidence for *C_D* and finally (iii) Closure holds yet again—if one has evidence for *B_D*, one has evidence for *C_D*.

2. Okasha on Wright In a recent paper, Samir Okasha [Okasha, 2004] sets out with the laudable aim to precisify and evaluate Wright’s claims within a plausible formal epistemological framework. His framework of choice is Bayesian confirmation theory, the most popular and arguably the most successful such framework on the market. His aim is to establish whether or not Exception, suitably translated into Bayesian terms, provides a sufficient condition for the failure of Transmission, again suitably translated into Bayesian terminology. Following Bayesian orthodoxy, Okasha first endorses

Evidence: For all propositions *A* and *B*, necessarily, *A* is evidence for *B* iff *A* raises the probability of *B*, i.e. iff $\Pr(B | A) > \Pr(B)$, where \Pr is interpreted as a function mapping propositions onto rational degrees of belief.

He moves on to formally translate Exception as

Exception*: A raises the probability of B conditional on C but not otherwise, i.e.
(1) $\Pr(B | A \cap C) > \Pr(B | C)$ and (2) $\Pr(B | A) \leq \Pr(B)$.

He then proves

Theorem: For all propositions A , B , and C , necessarily, if Exception* and $B \subseteq C$, then $\Pr(C | A) \leq \Pr(C)$.

And this, he says, provides a Bayesian vindication of Wright's claim that if 'the support that A gives to B is conditional on having independent reason to believe C ' then 'the support that A gives to B does not transmit to C , despite B entailing C '. But this is extremely puzzling: on Okasha's account of Exception, if Exception is true, then Transmission is *satisfied* rather than violated. Indeed, here is Okasha's own informal statement of Transmission (which agrees with my own):

[For all propositions A , B and C , necessarily,] if A is evidence for B and B entails C , then A will be evidence for C too.

This gives us, in conjunction with Evidence,

Transmission*: For all propositions A , B and C , necessarily, if (1) $\Pr(B | A) > \Pr(B)$ and (2) $B \subseteq C$, then (3) $\Pr(C | A) > \Pr(C)$.

So, by Theorem, sets of propositions for which Exception* is true do fail to satisfy the consequent of Transmission* ... but they also fail to satisfy its antecedent by satisfying condition (2) of Exception*: Transmission* is vacuously satisfied!

Furthermore, intuitively, Exception* is false of A_Z-C_Z . A_Z increases the probability of B_Z , whether or not one conditionalises on C_Z . In other words, whether or not one conditionalises on its not being a cleverly painted mule, the animal's being a black and white-striped four-legged creature presumably lowers the probability of a number of ways in which it could fail to be a zebra (e.g. its being a chicken), hence increases the probability of its being a zebra. It turns out that Okasha is in fact extremely careful to avoid claiming that cases like Zebra do in fact satisfy (1)–(3) of Transmission*. He explicitly chooses to stay neutral on the issue [Okasha, 2004, p. 143]. But of course, if Zebra clearly does not satisfy the conditions that Okasha takes Wright to claim it does, and one wants to remain charitable to Wright, this might further indicate that Okasha's interpretation of Wright is off the mark.

Before moving on, I would like to briefly address the suggestion made to me that a more charitable interpretation of Okasha's proposal may be available.² The alternative interpretation in question makes use of counterfactual conditionals involving probability statements. A Lewis/Stalnaker-style semantics can be straightforwardly provided for these, grounded in some kind of relevant ordering of the set of all probability functions, with \Pr (the actual probability function) as unique maximal element.³ The proposal is as follows. It is granted that Exception* is indeed false of the probability function \Pr characterising Zebra, i.e. that it is the case that $\Pr(B_Z | A_Z) > \Pr(B_Z)$.

However, the thought goes, ‘it may be that we are intended to interpret Exception* relative to some other probability function [Pr*], one that does not already take [C_Z] as given’, where C_Z is ‘taken as given’ iff Pr(C_Z) = 1, and Pr* is the probability function that would be actual were C_Z not to be taken as given. In other words, we should translate Exception as follows: were the probability of C to be strictly lesser than 1, A would raise the probability of B conditional on C but not otherwise, i.e. $\Pr(C) < 1 \quad \square \rightarrow \quad [\Pr(B | A \cap C) > \Pr(B | C) \wedge \Pr(B | A) \leq \Pr(B)]$.

However, setting aside the exegetical issue of whether or not this proposal is a plausible interpretation of Okasha’s account (this seems somewhat debatable), this suggestion does not fare any better than the one previously discussed. For one thing, it seems patently false that $\Pr(C_Z) < 1 \quad \square \rightarrow \quad \Pr(B_Z | A_Z) \leq \Pr(B_Z)$. Becoming less-than-certain that C_Z is true should not alter the fact that the animal’s being a black and white–striped four-legged creature lowers the probability of various alternatives to its being a zebra (e.g. its being a chicken) and hence increases the probability of its being a zebra. Furthermore, the probabilistic structure of Zebra is plausibly such that C_Z is *not* ‘taken as given’ (plausibly: $\Pr(C_Z) < 1$). If this is indeed the case, then Pr *itself* is the function most similar to Pr in which C_Z is not taken as given and hence the function with respect to which $\Pr(B | A \cap C) > \Pr(B | C) \wedge \Pr(B | A) \leq \Pr(B)$ is to be evaluated. But then Exception** is false of Zebra, as $\Pr(B_Z | A_Z) > \Pr(B_Z)$, whilst Wright claims that Exception is true of it.

3. An alternative Bayesian interpretation We have seen that Okasha’s suggestion is a non-starter. Is there a more promising proposal that could be made on behalf of the Bayesian? It would appear that there is. A first point that needs to be made is that Evidence does not capture what Wright had in mind. Indeed, from published comments, it is clear that by ‘evidence’ he means evidence that is sufficient for rational acceptability (confirmed in pers. comm.). How might this concept be translated into probabilistic terms? Here is a proposal:

Sufficient evidence: For all propositions A and B, necessarily, A is sufficient evidence for B iff A raises the probability of B and does so over some relevant probability-threshold $t \in [0, 1]$ sufficient for rational acceptability, i.e. iff $\Pr(B | A) > \Pr(B)$ and $\Pr(B | A) > t$.⁴

This suggests

Transmission:** For all propositions A, B, and C, necessarily, if (1a) $\Pr(B | A) > \Pr(B)$, (1b) $\Pr(B | A) > t$ and (2) $B \subseteq C$, then (3a) $\Pr(C | A) > \Pr(C)$ and (3b) $\Pr(C | A) > t$.⁵

Now Transmission** is false. There are cases of failure of Transmission, so defined, in which clauses (1a)–(3a) are satisfied but clause (3b) is not.⁶ This squares with Wright’s misgivings about Transmission. Furthermore, Transmission** seems to fail in precisely the kind of cases that Wright alludes to. Consider Zebra. Plausibly, $\Pr(C_Z | A_Z) < \Pr(C_Z)$. That the animal in the pen is a black and white–striped four-legged beast makes it more likely that it is a cleverly painted mule (e.g. by ruling out its being a

chicken) and hence less likely that it is not one. Again, Death Cap intuitively provides a case in which Transmission** succeeds: $\Pr(C_D | A_D) > \Pr(C_D)$.⁷

So much for giving Transmission the Bayesian treatment. What about Exception? The simplest interpretation would be the following:

Exception:** $\Pr(B | A) > t \supset \Pr(C) > t$.

In other words: either the support afforded by A is not sufficient for rational acceptance of B or C is already rationally acceptable.

However, Exception**, in conjunction with (1a), (1b) and (2) of Transmission** (i.e. assuming that there is entailment and evidential support to transmit in the first place), is *necessary* but unfortunately *insufficient* for transmission-failure.⁸ Of course, at this point, the Bayesian might perhaps complain that nowhere does Wright actually *establish* that Exception is sufficient for transmission-failure. What he does do is to offer various cases in which transmission fails and to note that Exception is true of them. The outcome of this procedure however, if anything, is to merely lend weight to the claim that Exception is a *consequence* of transmission-failure.

But it is also worth pointing out that the above analysis does vindicate *some* of what Wright has to say on the topic. Elsewhere in his writings, he tells us that ‘a transmissible warrant should make for the possible . . . overcoming of doubt or agnosticism’. This comes out true on the above suggestion, insofar as whilst it *is* rationally permissible to have a credence function such that warrant transmits and Exception is false (i.e. to be in a situation in which one is doubtful of C but would overcome this doubt upon finding out that A), it *is not* rationally permissible to have a credence function such that transmission fails and Exception is false.⁹

It has been suggested to me that there may be an alternative proposal that is arguably a more intuitive translation of Exception and *does* deliver a sufficient condition for transmission-failure. The idea is to try to secure sufficiency by moving from a material conditional to an appropriate counterfactual one, thereby strengthening Exception**. The idea would be to simply contrapose the material conditional in Exception** and swap the horseshoe for a box arrow, as follows: $\Pr(C) \leq t \square \rightarrow \Pr(B | A) \leq t$. In other words: were C not to be rationally acceptable, the support afforded by A would not be sufficient for rational acceptance of B .¹⁰ This is an interesting suggestion, and one which may be worth attempting to develop. However, assuming that Transmission** is indeed the correct translation of Transmission, until we are clearer on how to formally handle the counterfactual involved, it is not immediately obvious why, on this proposal, satisfaction of Exception would be sufficient for transmission-failure.¹¹

So, to recapitulate: we have failed to offer a probabilistic interpretation of Wright that *fully* vindicates his views, at least insofar as he is committed to the claim that Exception is sufficient for Transmission. We can however offer a plausible probabilistic characterisation of Transmission such that Transmission admits of exceptions, as required. Furthermore, intuitively, cases of transmission-failure such as Zebra have the probabilistic structure to indeed qualify as exceptions, on the analysis proposed, again as required. We also have a fairly natural translation of Exception, which is, if not sufficient for transmission-failure, at least necessary. Finally, the analysis vindicates Wright’s claim that transmission makes for the possible overcoming of doubt.

4. A complication There is however one last issue to be addressed: it turns out that Wright would think that the consequent of Transmission** is too weak.¹² In other words, according to him there are ways of achieving transmission-failure besides making the consequent of Transmission** false. Indeed, he discusses a further alleged case of transmission-failure, besides Zebra-style cases, in which it is plausible that the evidence considered in support of the premise is also sufficient evidence for the conclusion. Here is the example [Wright, 2002]:

Election: A_E : Jones has just placed an ‘X’ on a ballot paper. B_E : Jones has just voted.
 C_E : An election is taking place.

Although, in standard circumstances, A_E is sufficient evidence for C_E , the nuance here is that A_E is not sufficient evidence for C_E because it is evidence for B_E ; rather, Wright tells us, it is sufficient evidence for C_E only because it is already sufficient *direct* evidence for C_E : were it not to be direct evidence for C_E , it would cease to be evidence for C_E altogether. For transmission to succeed, we are told, it must be the case that were the evidence supporting the premises to cease to be sufficient *direct* evidence for the conclusion, it would nevertheless still be sufficient evidence therefor.

What exactly does Wright mean by this? We first need to get clear on what it is for A_E to support C_E directly (or perhaps more precisely: for A_E to support C_E in a manner that is unmediated by B ; it may indeed be mediated by other propositions). Wright hints at a criterion in the context of his discussion of an example structurally similar to Election:

Football: A_F : Jones has just kicked the ball between the white posts. B_F : Jones has just scored a goal. C_F : A game of football is taking place.

A_F , he tells us, is direct evidence for C_F because

we would have had... warrant for [C_F], on the basis of the very same evidence [i.e. A_F], even if we had also noticed the referee’s assistant’s flag raised to mark an infringement and reckoned it quite likely that the defending team would direct the referee’s attention to that fact in the next few seconds. [Wright, 2002, p. 334]

Presumably, to notice the referee’s assistant’s flag being raised, etc. amounts to coming to know that $\overline{B_F}$, i.e. that Jones has not scored a goal. So one might suggest the following general definition:

Sufficient direct evidence: For all propositions A , B , and C , necessarily, A is sufficient direct evidence for C with respect to B , i.e. sufficient evidence for C unmediated by B , iff (i) $\Pr(C | A \cap \overline{B}) > \Pr(C | \overline{B})$ and (ii) $\Pr(C | A \cap \overline{B}) > t$.

It is plausible that (i) and (ii) of this definition are both true of A_E , B_E and C_E in Election, as well as A_F , B_F and C_F in Football, at least in standard circumstances. It is also plausible that (i) and (ii) are both false of A_D , B_D and C_D in Death Cap. In the light of this, we can therefore offer the following formal account of Transmission:

Transmission*:** For all propositions A , B , and C , necessarily, if (1a) $\Pr(B | A) > \Pr(B)$, (1b) $\Pr(B | A) > t$ and (2) $B \subseteq C$, then (3a) $\Pr(C | A) > \Pr(C)$, (3b) $\Pr(C | A) > t$ and (3c) $\Pr(C | A \cap \overline{B}) < t \quad \square \rightarrow \Pr(C | A) > t$.

Clause (3c) aims to capture the thought that were the *direct* part of the support afforded by A not sufficient for rational acceptability of C , the support offered by A would no longer be sufficient either. Death Cap clearly satisfies Transmission***, as (1a) to (3b) are obviously satisfied and (3c) holds simply because, as a matter of actual fact, $\Pr(C_D | A_D \cap \overline{B_D}) < t$. It is furthermore plausible that it is violated in both Election and Football.

In support of this last intuition, Wright [Wright, 2002, p. 334] asks us to consider a society in which election drills are held as often as the real deal, election drills being such that we would fail to discriminate them from the genuine article on the basis of our sole observation of Jones and his surrounding circumstances. In these circumstances, he tells us, A_E no longer provides sufficient direct evidence for C_E and provides insufficient evidence for either B_E or C_E .

Now of course, this putative fact about the properties of ‘drill’ societies falls short of establishing the counterfactual that Wright is after (i.e. the counterfactual that we have translated as (3c)). Whilst drill societies do provide a case in which A_E is neither sufficient direct evidence for C_E , nor sufficient evidence for either B_E or C_E , it is not immediately obvious that in ‘normal’ societies, were A_E not to be sufficient direct evidence for C_E , we would wind up with something having the pertinent probabilistic properties of a drill society. Still, pending an account of what *would* be the case were A_E to cease to be sufficient direct evidence for C_E , it is interesting to note that Wright’s claims regarding drill societies appear to be borne out formally in our Bayesian framework.

Let \Pr_N denote the probability distribution pertinent to normal societies such as our own, in which the prior probability of a drill is vanishingly small, and let \Pr_D denote the probability distribution pertinent to drill societies, which are as similar to normal societies as is allowed by the fact that drills are as frequent there as *bona fide* elections. The claim that A_E does not, in either normal or drill societies, allow one to discriminate between the fact that there is an election taking place (C_E) and the fact that there is an election drill taking place (C_E^2) can be satisfactorily translated as the conjunction of $\Pr_N(A_E | C_E) = \Pr_N(A_E | C_E^2)$ and $\Pr_D(A_E | C_E) = \Pr_D(A_E | C_E^2)$.¹³

Intuitively, \Pr_D is obtained from \Pr_N , and *vice versa*, by simply making the most conservative adjustments necessary to accommodate a shift in the relative probabilities of C_E and various alternatives thereto, including C_E^2 . Specifically, \Pr_D (\Pr_N) is plausibly obtained from \Pr_N (\Pr_D) by increasing (lowering) the probability of C_E^2 in relation to the probability of C_E , whilst minimising the impact of this shift on the remainder of the function. It is well known that on the most plausible measures of distance between probability distributions, such as the Kullback–Leibler distance, this procedure will have to amount to what is known as ‘Jeffrey-conditionalisation’ with respect to some partition $\{C_E^i\}$ that includes C_E and C_E^2 .¹⁴ In particular, we move from \Pr_N to \Pr_D by Jeffrey-conditionalising so as to obtain $\Pr_D(C_E) = \Pr_D(C_E^2)$.

But now we can easily prove the following, assuming that $t > 0.5$ (an assumption that would appear to be shared by Wright): if we obtain \Pr_D from \Pr_N by Jeffrey-

conditionalising with respect to $\{C_E^i\}$ so that $\Pr_D(C_E) = \Pr_D(C_E^2)$, then $\Pr_D(C_E | A_E)$, $\Pr_D(B_E | A_E)$ and $\Pr_D(C_E | A_E \cap \overline{B_E})$ are all $< t$.¹⁵ Wright's views on the probabilistic structure of drill societies come out vindicated: on the present analysis, in such societies A_E no longer provides sufficient direct evidence for C_E and provides insufficient evidence for either B_E or C_E , as conjectured.

5. Okasha on Election To wrap things up, a brief discussion of Okasha's own treatment of the Election case is in order. After presenting the Bayesian analysis of transmission-failure sketched out in section 2 above, Okasha does take note of Wright's discussion of Election, but remains perplexed. He complains:

our proof above [(i.e. the proof of Theorem)] shows that, where A supports B , but only conditional on C , then $\Pr(C | A) \leq \Pr(C)$, hence A does not support C , directly or indirectly. But Wright wants to say that in the voting example... A supports C directly... Of course Wright could argue that (i) $[\Pr(B | A \cap C) > \Pr(B | C)]$ and (ii) $[\Pr(B | A) \leq \Pr(B)]$ do not adequately capture his idea that A supports B only if there is independent reason to believe C but this move seems ad hoc; (i) and (ii) are very natural probabilistic translations of this idea. [Okasha, 2004, p. 144]

He concludes that 'Wright's distinction between direct and indirect support muddies the waters'. Urging us to 'abandon, or ignore, [the] distinction', he moves on to (i) reject Wright's claim that transmission fails in Election-normal society and (ii) endorse Wright's claim that transmission fails in Election-drill society, diagnosing the difference between the two cases as follows. In normal societies, he tells us, the following hold:

1. $\Pr_N(B_E | A_E \cap C_E) > \Pr_N(B_E | C_E)$,
2. $\Pr_N(B_E | A_E) > \Pr_N(B_E)$,
3. $\Pr_N(C_E | A_E) > \Pr_N(C_E)$.¹⁶

In contrast, drill societies are supposedly characterised by

1. $\Pr_D(B_E | A_E \cap C_E) > \Pr_D(B_E | C_E)$,
2. $\Pr_D(B_E | A_E) \leq \Pr_D(B_E)$,
3. $\Pr_D(C_E | A_E) \leq \Pr_D(C_E)$.

In other words, in normal societies, A_E boosts the probabilities of both B_E and C_E , whilst in drill societies, it boosts the probabilities of neither. Hence, according to Okasha's understanding of Transmission (i.e. Transmission*), transmission succeeds in the former case and fails in the latter.

In relation to this, two comments. First of all, with respect to the passage quoted a few paragraphs up, there is no reason why Wright should say that the support afforded by A_E would only be sufficient for rational acceptance of B_E on condition that C_E is

already rationally acceptable prior to finding out the truth of A_E (and as far as I can tell, he is not saying this). After all, Wright only claims that Exception is *sufficient* for transmission-failure. He leaves it open whether or not it is also *necessary*. So even if (i) and (ii) were ‘natural probabilistic translations of this idea [that A supports B only if there is independent reason to believe C]’, this need not be relevant to the case at hand. Transmission fails, in this case, not due to the fact that A_E does not support C_E , but due to the fact that A_E supports C_E only directly. Secondly, Okasha’s view of the probabilistic structure of normal versus drill societies is simply implausible. If Pr_D is indeed obtained from Pr_N , and *vice versa*, by Jeffrey-conditionalisation with respect to $\{C_E^i\}$, as suggested in the previous section, then A_E boosts the probabilities of both B_E and C_E in normal societies *iff it does so in drill societies*.¹⁷ And indeed, setting Jeffrey-conditionalisation aside, the probabilistic structure of drill societies is intuitively such that, just as in normal societies, A_E *does* boost the probabilities of B_E as well as C_E . The scene involving Jones presumably makes it more likely that there is an election being held (of course, it *also* makes it more likely that there is a drill being held, but that is beside the point). It also presumably makes it more likely that Jones has just voted (again, it also makes it more likely that he has just pretend-voted, but that is also irrelevant).

Notes

¹Wright sometimes frames his discussion in terms of transmission of ‘warrant’ rather than ‘evidential support’. This may be a more general formulation: arguably, one can have warrant for a proposition P without this warrant being due to the possession of an item of evidence that evidentially supports P . However, as Okasha notes [Okasha, 2004, p. 139], to the extent that some warrant has evidential underpinnings, some failures of transmission of evidential support will lead to failures of transmission of warrant.

²The issue was raised in an anonymous referee report.

³There are of course a number of plausible ways of doing this, but one attractive possibility might be to base this ordering on relative distance from Pr according to some suitable distance metric, such as Kullback–Leibler distance.

⁴For present purposes, we can leave open the issue of whether or not t should be taken to be context-dependent or not, if so how context determines its value and if not, whether it has unit or sub-unit value. Those Bayesians who hold that the acceptability of a proposition supervenes on its probability (a number do not, e.g. [Maher, 1993], and I would tend to agree with them), typically hold that $0.5 < t < 1$ across all contexts.

⁵Note that Closure can be plausibly translated as

Closure*: If $\text{Pr}(B | A) > t$ and $B \subseteq C$, then $\text{Pr}(C | A) > t$,

which, being a well-known theorem of the probability calculus, would also vindicate Wright’s aforementioned suspicions that Closure is true.

⁶To illustrate, here is such model, for $t = 0.9$: $\text{Pr}(A \cap B \cap \bar{C}) = \text{Pr}(\bar{A} \cap B \cap \bar{C}) = \text{Pr}(\emptyset) = 0$, $\text{Pr}(A \cap B \cap C) = 0.09$, $\text{Pr}(A \cap \bar{B} \cap C) = 0.002$, $\text{Pr}(A \cap \bar{B} \cap \bar{C}) = 0.008$, $\text{Pr}(\bar{A} \cap B \cap C) = 0.05$, $\text{Pr}(\bar{A} \cap B \cap \bar{C}) = 0.8$ and $\text{Pr}(\bar{A} \cap \bar{B} \cap \bar{C}) = 0.05$. Analogous models demonstrably exist for any value of $t \in [0, 1]$. Note that the conjunction of (1b) and (2) entails (3b), so transmission-failure will have to take this form (rather than failing by (3b) coming out false). That Transmission** is false should not really come as a surprise; it is virtually identical to the Bayesian translation of the so-called ‘Special Consequence Condition’ [Hempel, 1945, p. 103], which was noted to be false quite some time ago.

⁷In fact, Zebra clearly satisfies the following sufficient condition for failure of Transmission, as defined by Transmission**: $\text{Pr}(A_Z | \bar{C}_Z) = \text{Pr}(A_Z | B_Z)$. Interestingly, according to the so-called ‘Law of Likelihood’ (defended notably by [Hacking, 1964] and more recently by [Royall, 1997]), this condition is both necessary and sufficient for A_Z to count as failing to favour \bar{C}_Z over B_Z .

⁸*Proof:* It is necessary because, obviously, as $B \subseteq C$, we have $\Pr(B | A) < \Pr(C | A)$, so if $\Pr(C | A) < \Pr(C)$ and $\Pr(B | A) > t$, then $\Pr(C) > t$. However, the model given above in footnote 6 demonstrates a failure of sufficiency, for $t = 0.9$. Analogous models demonstrably exist for any value of $t \in [0, 1]$.

⁹Compare: good weather makes for the possibility of a pleasant picnic insofar as it is possible to have a pleasant picnic in the sun but not possible to have a pleasant picnic in the rain.

¹⁰The original suggestion, which I owe to a perspicuous anonymous referee, was in fact the slightly weaker $\Pr(C) \leq t \Leftrightarrow \neg[\Pr(B | A) > t > \Pr(B)]$.

¹¹Note that moving to a ‘strict conditional’, requiring that that $\Pr(B | A) > t \supset \Pr(C) > t$ for all rationally permissible credence functions, would not help here. Indeed, according to Bayesian orthodoxy, all that it takes for a credence function to be rationally permissible is that it be a probability function. In this case, Exception would come out as a necessary falsehood and *would* be sufficient for transmission-failure, albeit at the cost of being so in a trivial sense.

¹²In fact, he has informed me in conversation that he also holds that it is too *strong*, on the grounds that he takes issue with the widespread claim that evidence must raise the probability of that which it is evidence for, much in the same way that some argue against the claim that causes must raise the probability of their effects (see for instance [Hesslow, 1976] and [Rosen, 1978]). Although I do believe that the cases that he raises in support of this claim can be dealt with without any real difficulty, I shall have to sidestep this issue for now.

¹³See footnote 7 above.

¹⁴In other words, the procedure will be such that for all D_E , $\Pr_D(D_E) = \sum \Pr_N(D_E | C_E^i) \Pr_D(C_E^i)$ and $\Pr_N(D_E) = \sum \Pr_D(D_E | C_E^i) \Pr_N(C_E^i)$.

¹⁵*Proof:* $\Pr_D(A | C) = \Pr_D(A | C^2)$ (by assumption). If we assume that $\Pr_D(C) = \Pr_D(C_E^2)$, it follows that $\Pr_D(C | A) = \Pr_D(C^2 | A)$. Given that $C \cap C^2 = \emptyset$, $\Pr_D(C | A) \leq 0.5 < t$, and hence $\Pr_D(B | A) < t$. Now $\Pr_D(C | A) = \Pr_D(C | A \cap B) \Pr_D(B | A) + \Pr_D(C | A \cap \bar{B}) \Pr_D(\bar{B} | A) = \Pr_D(B | A) + \Pr_D(C | A \cap \bar{B}) \Pr_D(\bar{B} | A)$ (axioms of prob.). Furthermore $\Pr_D(C^2 | A) = \Pr_D(C^2 | A \cap B) \Pr_D(B | A) + \Pr_D(C^2 | A \cap \bar{B}) \Pr_D(\bar{B} | A) = \Pr_D(C^2 | A \cap \bar{B}) \Pr_D(\bar{B} | A)$ (axioms of prob.). So $\Pr_D(B | A) + \Pr_D(C | A \cap \bar{B}) \Pr_D(\bar{B} | A) = \Pr_D(C^2 | A \cap \bar{B}) \Pr_D(\bar{B} | A)$. It follows that $\Pr_D(C | A \cap \bar{B}) \leq \Pr_D(C^2 | A \cap \bar{B})$, hence $\Pr_D(C | A \cap \bar{B}) < t$.

¹⁶Note that, whilst I use the subscripts N and D to distinguish between the probability functions characterising, respectively, normal and drill societies, Okasha marks the distinction by conditionalising the same probability function on two different corpuses of knowledge. Of course, absolutely nothing of substance hinges on this.

¹⁷*Proof:* $\Pr_N(C_E | A_E) > \Pr_N(C_E)$ iff $\Pr_N(A_E | C_E) > \Pr_N(A_E | \bar{C}_E)$. But if \Pr_D is obtained from \Pr_N , and *vice versa*, by Jeffrey-conditionalising with respect to $\{C_E^i\}$, then $\Pr_N(A_E | C_E) > \Pr_N(A_E | \bar{C}_E)$ iff $\Pr_D(A_E | C_E) > \Pr_D(A_E | \bar{C}_E)$, as Jeffrey-conditionalising with respect to a given partition leaves unchanged the probabilities of any event conditional on the various cells of that partition. And of course, $\Pr_D(A_E | C_E) > \Pr_D(A_E | \bar{C}_E)$ iff $\Pr_D(C_E | A_E) > \Pr_D(C_E)$. Furthermore, on the assumption that $\Pr_N(B_E | A_E \cap C_E) > \Pr_N(B_E | C_E)$ and $\Pr_D(B_E | A_E \cap C_E) > \Pr_D(B_E | C_E)$, it then also follows from Theorem that $\Pr_N(B_E | A_E) > \Pr_N(B_E)$ iff $\Pr_D(B_E | A_E) > \Pr_D(B_E)$.

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